AP Calculus
SUMMER REVIEW PACKET

For students entering ***CALCULUS***

Name: ____

Course: ______________________ Period ___________________

The problems in this packet are designed to help you review topics from Algebra and pre-calculus which are important to your success in AP Calculus. All too often, I have seen students show beautiful calculus work, only to get the final answer incorrect due to an algebra error, for example. SO… do yourself a favor and work out these pre-requisite skills at your leisure during the summer. When you come across a topic that requires a little review, feel free to search a website, or call up a friend for help.

For AP Calc AB and BC Students: Bring in this packet (showing work!) due on the first day of class. Yes, this will be incorporated as a required grade for Quarter 1. Also, review Unit Circle (included). Make copies of the Unit Circle Worksheet and practice repeatedly until you are very familiar with trig functions of these basic angles. **Be prepared for a short no calculator quiz on trig functions of basic angles 2nd day of class.**

Complex Fractions

When simplifying complex fractions, multiply by a fraction equal to 1 which has a numerator and denominator composed of the common denominator of all the denominators in the complex fraction.

Example:

$$\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} = \frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} \bullet \frac{x+1}{x+1} = \frac{-7(x+1)-6}{5} = \frac{-7x-13}{5}$$

$$\frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} = \frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} \bullet \frac{x(x-4)}{x(x-4)} = \frac{-2(x-4)+3x(x)}{5(x)(x-4)-1(x)} = \frac{-2x+8+3x^2}{5x^2-20x-x} = \frac{3x^2-2x+8}{5x^2-21x}$$

Simplify each of the following.

1. $\dfrac{\frac{25}{a}-a}{5+a}$

2. $\dfrac{2-\frac{4}{x+2}}{5+\frac{10}{x+2}}$

3. $\dfrac{4-\frac{12}{2x-3}}{5+\frac{15}{2x-3}}$

4. $\dfrac{\frac{x}{x+1}-\frac{1}{x}}{\frac{x}{x+1}+\frac{1}{x}}$

5. $\dfrac{1-\frac{2x}{3x-4}}{x+\frac{32}{3x-4}}$

Simplifying Expressions

Make sure you are very comfortable manipulating exponents, positive/negative, fractional. Also, know the relationship between exponents and radicals; the appropriate radical will "undo" an exponent.

Examples:

$\sqrt[3]{(2)^3} = 2$

$\sqrt[3]{8} = 8^{1/3} = 2$

$\left(\sqrt[10]{25}\right)^5 = \left(25^{1/10}\right)^5 = 25^{5/10} = 25^{1/2} = \sqrt{25} = 5$

$x^{-4} = \dfrac{1}{x^4}$

$\dfrac{1}{x^{-3}} = x^3$

$(3x)^{-2} = \dfrac{1}{(3x)^2} = \dfrac{1}{9x^2}$

$x^6 x^5 = x^{11}$

$\dfrac{x^3}{x^9} = x^{-6} = \dfrac{1}{x^6}$

$\left(x^2\right)^3 = x^6$

Simplify each expression. Write answers with positive exponents where applicable.

A. $$\frac{1}{x+h} - \frac{1}{x}$$

B. $$\frac{\dfrac{2}{x^2}}{\dfrac{10}{x^3}}$$

C. $$\frac{12x^{-3}y^2}{18xy^{-1}}$$

D. $$\frac{15x^2}{5\sqrt{x}}$$

E. $$\left(5a^3\right)\left(4a^2\right)$$

F. $$\left(4a^{\frac{5}{3}}\right)^{\frac{3}{2}}$$

G. $$\frac{\dfrac{1}{2} - \dfrac{5}{4}}{\dfrac{3}{8}}$$

H. $$\frac{5-x}{x^2-25}$$

Functions

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "***f*** of ***g*** of ***x***" means: plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let $f(x) = 2x + 1$ *and* $g(x) = 2x^2 - 1$**. Find each.**

6. $f(2) =$ ____________
7. $g(-3) =$ _____________
8. $f(t+1) =$ __________
9. $f\left[g(-2)\right] =$ __________
10. $g\left[f(m+2)\right] =$ ___________
11. $\dfrac{f(x+h) - f(x)}{h} =$ ______

Let $f(x) = \sin x$ **Find each exactly.**

12. $f\left(\dfrac{\pi}{2}\right) =$ ___________
13. $f\left(\dfrac{2\pi}{3}\right) =$ ______________

Let $f(x) = x^2$, $g(x) = 2x + 5$, *and* $h(x) = x^2 - 1$**. Find each.**

14. $h\left[f(-2)\right] =$ _______
15. $f\left[g(x-1)\right] =$ _______
16. $g\left[h(x^3)\right] =$ _______

Find $\frac{f(x+h)-f(x)}{h}$ **for the given function** ***f.***

17. $f(x)=9x+3$

18. $f(x)=5-2x$

Intercepts and Points of Intersection

To find the x-intercepts, let y = 0 in your equation and solve.
To find the y-intercepts, let x = 0 in your equation and solve.

Example: $y=x^2-2x-3$

$\underline{x-\text{int}.\ (Let\ y=0)}$

$0=x^2-2x-3$

$0=(x-3)(x+1)$

$x=-1\ or\ x=3$

$x-\text{intercepts}\ (-1,0)\ and\ (3,0)$

$\underline{y-\text{int}.\ (Let\ x=0)}$

$y=0^2-2(0)-3$

$y=-3$

$y-\text{intercept}\ (0,-3)$

Find the x and y intercepts for each.

19. $y=2x-5$

20. $y=x^2+x-2$

21. $y=x\sqrt{16-x^2}$

22. $y^2=x^3-4x$

Use substitution or elimination method to solve the system of equations.
Example:

$$x^2 + y^2 - 16x + 39 = 0$$
$$x^2 - y^2 - 9 = 0$$

Elimination Method

$2x^2 - 16x + 30 = 0$

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3$ *and* $x = 5$

Plug $x = 3$ *and* $x = 5$ into one original

$3^2 - y^2 - 9 = 0$ $\qquad$ $5^2 - y^2 - 9 = 0$

$-y^2 = 0$ $\qquad$ $16 = y^2$

$y = 0$ $\qquad$ $y = \pm 4$

Points of Intersection $(5,4)$, $(5,-4)$ *and* $(3,0)$

Substitution Method

Solve one equation for one variable.

$y^2 = -x^2 + 16x - 39$ (1st equation solved for y)

$x^2 - (-x^2 + 16x - 39) - 9 = 0$ Plug what y^2 is equal to into second equation.

$2x^2 - 16x + 30 = 0$ (*The rest is the same as*

$x^2 - 8x + 15 = 0$ *previous example*)

$(x-3)(x-5) = 0$

$x = 3$ *or* $x - 5$

Find the point(s) of intersection of the graphs for the given equations.

23. $\begin{aligned} x + y &= 8 \\ 4x - y &= 7 \end{aligned}$

24. $\begin{aligned} x^2 + y &= 6 \\ x + y &= 4 \end{aligned}$

25. $\begin{aligned} 2x - 3y &= 5 \\ 5x - 4y &= 6 \end{aligned}$

<u>Interval Notation</u>

26. Complete the table with the appropriate notation or graph.

Solution	Interval Notation	Graph
$-2 < x \leq 4$		
	$[-1, 7)$	
		

Solve each equation. State your answer in BOTH interval notation and graphically.

27. $2x-1\geq 0$

28. $-4\leq 2x-3<4$

29. $\frac{x}{2}-\frac{x}{3}>5$

<u>Domain and Range</u>

Find the domain and range of each function. Write your answer in INTERVAL notation.

30. $f(x)=x^2-5$

31. $f(x)=-\sqrt{x+3}$

32. $f(x)=3\sin x$

33. $f(x)=\frac{2}{x-1}$

<u>Inverses</u>

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.
Example:

$f(x)=\sqrt[3]{x+1}$	Rewrite f(x) as y
$y=\sqrt[3]{x+1}$	Switch x and y
$x=\sqrt[3]{y+1}$	Solve for your new y
$(x)^3=\left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3=y+1$	Simplify
$y=x^3-1$	Solve for y
$f^{-1}(x)=x^3-1$	Rewrite in inverse notation

Find the inverse for each function.

34. $f(x)=2x+1$

35. $f(x)=\frac{x^2}{3}$

Also, recall that to PROVE one function is an inverse of another function, you need to show that: $f(g(x)) = g(f(x)) = x$

Example:

If: $f(x) = \dfrac{x-9}{4}$ *and* $g(x) = 4x+9$ **show *f(x) and g(x)* are inverses of each other.**

$$f(g(x)) = 4\left(\frac{x-9}{4}\right)+9 = x-9+9 = x$$

$$g(f(x)) = \frac{(4x+9)-9}{4} = \frac{4x+9-9}{4} = \frac{4x}{4} = x$$

$f(g(x)) = g(f(x)) = x$ therefore they are inverses of each other.

Prove *f and g* are inverses of each other.

36. $f(x) = \dfrac{x^3}{2}$ $\quad g(x) = \sqrt[3]{2x}$

37. $f(x) = 9 - x^2,\ x \geq 0$ $\quad g(x) = \sqrt{9-x}$

Equation of a line

Slope intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

Vertical line: x = c (slope is undefined)

Horizontal line: y = c (slope is 0)

38. Use slope-intercept form to find the equation of the line having a slope of 3 and a y-intercept of 5.

39. Determine the equation of a line passing through the point (5, -3) with an undefined slope.

40. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.

41. Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of 2/3.

42. Find the equation of a line passing through the point (2, 8) and parallel to the line $y = \frac{5}{6}x - 1$.

43. Find the equation of a line perpendicular to the y- axis passing through the point (4, 7).

44. Find the equation of a line passing through the points (-3, 6) and (1, 2).

45. Find the equation of a line with an x-intercept (2, 0) and a y-intercept (0, 3).

Radian and Degree Measure

To convert radian to degrees, make use of the fact that π radian equals one half of a circle, or 180˚. SO, divide radians by π, and you'll get the # of half circle, Multiply by 180 will give you the answer in degrees.

$$degrees = radians \times \frac{180}{\pi}$$

To convert degrees to radians, first find the # of half circles in the answer by dividing by 180˚. Since each half circle equals π radians, multiply the # of half circles by π get the answer in radian.

$$radians = degrees \times \frac{\pi}{180}$$

46. Convert to degrees: a. $\frac{5\pi}{6}$ b. $\frac{4\pi}{5}$ c. 2.63 radians

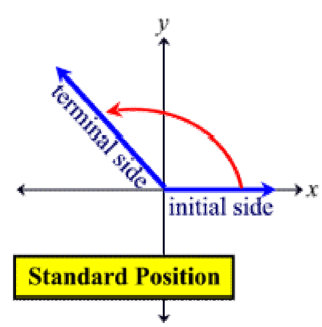
47. Convert to radians: a. 45° b. -17° c. 237°

Angles in Standard Position

Angle is in standard position if its vertex is located at the origin and one ray is on the positive x-axis (initial side). Other ray is the terminal side. Angle is measured by the amount of rotation from the initial side to the terminal side.

48. Sketch the angle in standard position.

a. $\frac{11\pi}{6}$ b. 230° c. $-\frac{5\pi}{3}$ d. 1.8 radians



<u>Reference Triangles</u> (If you don't remember how to do this, google "reference triangles")

49. Sketch the angle in standard position. Draw the reference triangle and label the sides, if possible.

a. $\frac{2}{3}\pi$

b. 225°

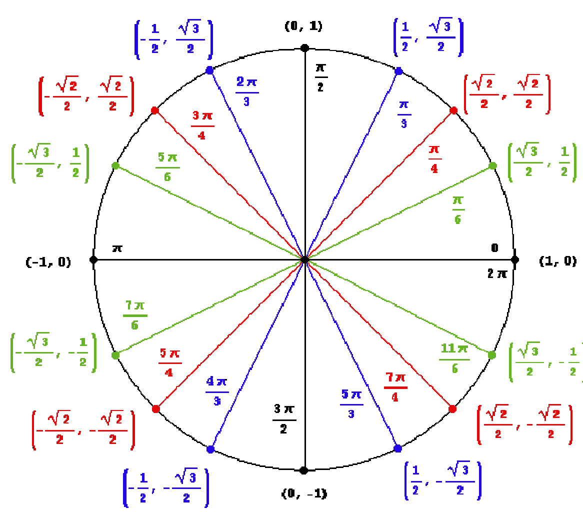
c. $-\frac{\pi}{4}$

d. 30°

<u>Unit Circle</u>

You can determine the sine or cosine for basic angles by using the unit circle. The x-coordinate of the circle is the cosine and the y-coordinate is the sine of the angle. See Unit Circle worksheet for more practice.

Example: $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ $\cos\frac{4\pi}{3} = -\frac{1}{2}$



50. $a.)\ \sin\pi =$

$b.)\ \cos\frac{5\pi}{4}$

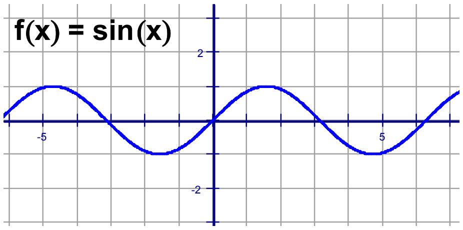
$c.)\ \sin(-\frac{\pi}{2}) =$

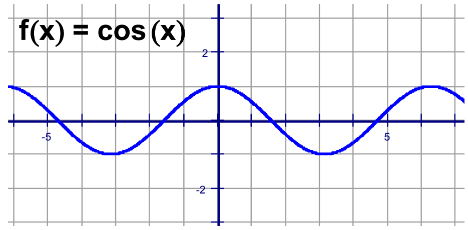
$d.)\ \sin\frac{11\pi}{6} =$

$e.)\ \cos 2\pi =$

$f.)\ \cos(-\pi) =$

Graphing Trig Functions





y = sin x and y = cos x have a period of 2π and an amplitude of 1. Use the parent graphs above to help you sketch a graph of the functions below. For $f(x) = A\sin(Bx + C) + K$, A = amplitude, $\frac{2\pi}{B}$ = period, $\frac{C}{B}$ = phase shift (positive C/B shift left, negative C/B shift right) and K = vertical shift.

Graph two complete periods of the function.

51. $f(x) = 5\sin x$

52. $f(x) = \sin 2x$

53. $f(x) = -\cos\left(x - \frac{\pi}{4}\right)$

54. $f(x) = \cos x - 3$

Using the Graphing Calculator

Spend some time getting comfortable with your graphing calculator, if you are not already. For calculus, you should be comfortable (1) graphing an arbitrary function, (2) finding zeros (or roots) of a function, (3) and finding the intersection between 2 functions.

55. Given: $f(x) = x^4-3x^3+2x^2-7x-11$
    Find all roots to the nearest 0.001

    [You can solve for f(x)=0 by graphing f(x), then going to Calc menu and #2 zero function]

56. Given: $f(x) = 3\sin 2x - 4x + 1$ from $[-2\pi, 2\pi]$
    Find all roots to the nearest 0.001
    [NOTE: all trig functions are done in radian mode]

57. Given $f(x) = 0.7x^2 + 3.2x + 1.5$
    Find all roots to the nearest 0.001

58. Given: $f(x) = x^4 - 8x^2 + 5$
    Find all roots to the nearest 0.001

59. Given $f(x) = x^2 - 5x + 2$ and $g(x) = 3 - 2x$
    Find the coordinates of any points of intersection

Limits

Finding limits numerically. (optional)

Complete the table and use the result to estimate the limit. (Optional, limit problems are NOT REQUIRED)

60. $\lim_{x\to 4} \frac{x-4}{x^2-3x-4}$

x	3.9	3.99	3.999	4.001	4.01	4.1
f(x)						

61. $\lim_{x\to -5} \frac{\sqrt{4-x}-3}{x+5}$

x	-5.1	-5.01	-5.001	-4.999	-4.99	-4.9
f(x)						

Finding limits graphically. (optional)

Find each limit graphically. Use your calculator to assist in graphing.

62. $\lim_{x\to 0} \cos x$

63. $\lim_{x\to 5} \frac{2}{x-5}$

64. $\lim_{x\to 1} f(x)$

$$f(x) = \begin{cases} x^2+3, & x \neq 1 \\ 2, & x = 1 \end{cases}$$

Evaluating Limits Analytically (optional)

Solve by direct substitution whenever possible. If needed, rearrange the expression so that you can do direct substitution.

65. $\lim_{x\to 2}(4x^2+3)$

66. $\lim_{x\to 1} \frac{x^2+x+2}{x+1}$

67. $\lim_{x\to 0} \sqrt{x^2+4}$

68. $\lim_{x\to \pi} \cos x$

69. $\lim_{x\to 1}\left(\frac{x^2-1}{x-1}\right)$ HINT: Factor and simplify.

70. $\lim_{x\to -3}\frac{x^2+x-6}{x+3}$

71. $\lim_{x\to 0}\frac{\sqrt{x+1}-1}{x}$ HINT: Multiply numerator & denominator by the conjugate.

72. $\lim_{x\to 3}\frac{3-x}{x^2-9}$

73. $\lim_{h\to 0}\frac{2(x+h)-2x}{h}$

<u>One-Sided Limits</u> (optional)

Find the limit if it exists. First, try to solve for the overall limit. If an overall limit exists, then the one-sided limit will be the same as the overall limit. If not, use the graph and/or a table of values to evaluate one-sided limits.

74. $\lim_{x\to 5^+}\frac{x-5}{x^2-25}$

75. $\lim_{x\to -3^-}\frac{x}{\sqrt{x^2-9}}$

76. $\lim_{x\to 10^+}\frac{|x-10|}{x-10}$

77. $\lim_{x\to 5^-}\left(-\frac{3}{x+5}\right)$

Vertical Asymptotes

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote.

78. $f(x)=\frac{1}{x^2}$

79. $f(x)=\frac{x^2}{x^2-4}$

80. $f(x)=\frac{2+x}{x^2(1-x)}$

Horizontal Asymptotes

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is $y = 0$.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Determine all Horizontal Asymptotes.

81. $f(x)=\frac{x^2-2x+1}{x^3+x-7}$

82. $f(x)=\frac{5x^3-2x^2+8}{4x-3x^3+5}$

83. $f(x)=\frac{4x^5}{x^2-7}$

Determine each limit as x goes to infinity.

RECALL: This is the same process you used to find Horizontal Asymptotes for a rational function.
**** In a nutshell**

1. Find the highest power of x.
2. How many of that type of x do you have in the numerator?
3. How many of that type of x do you have in the denominator?
4. If highest power in numerator = highest power in denominator, that ratio is your limit!

84. $\lim_{x\to\infty}\left(\frac{2x-5+4x^2}{3-5x+x^2}\right)$

85. $\lim_{x\to\infty}\left(\frac{2x-5}{3-5x+3x^2}\right)$

86. $\lim_{x\to\infty}\left(\frac{7x+6-2x^3}{3+14x+x^2}\right)$

Limits to Infinity (Optional)

A rational function does not have a limit if it goes to $\pm\infty$, however, you can state the direction the limit is headed if both the left and right hand side go in the same direction.

Determine each limit if it exists. If the limit approaches ∞ *or* $-\infty$, please state which one the limit approaches.

87. $\lim_{x \to -1^+} \frac{1}{x+1} =$

88. $\lim_{x \to 1^+} \frac{2+x}{1-x} =$

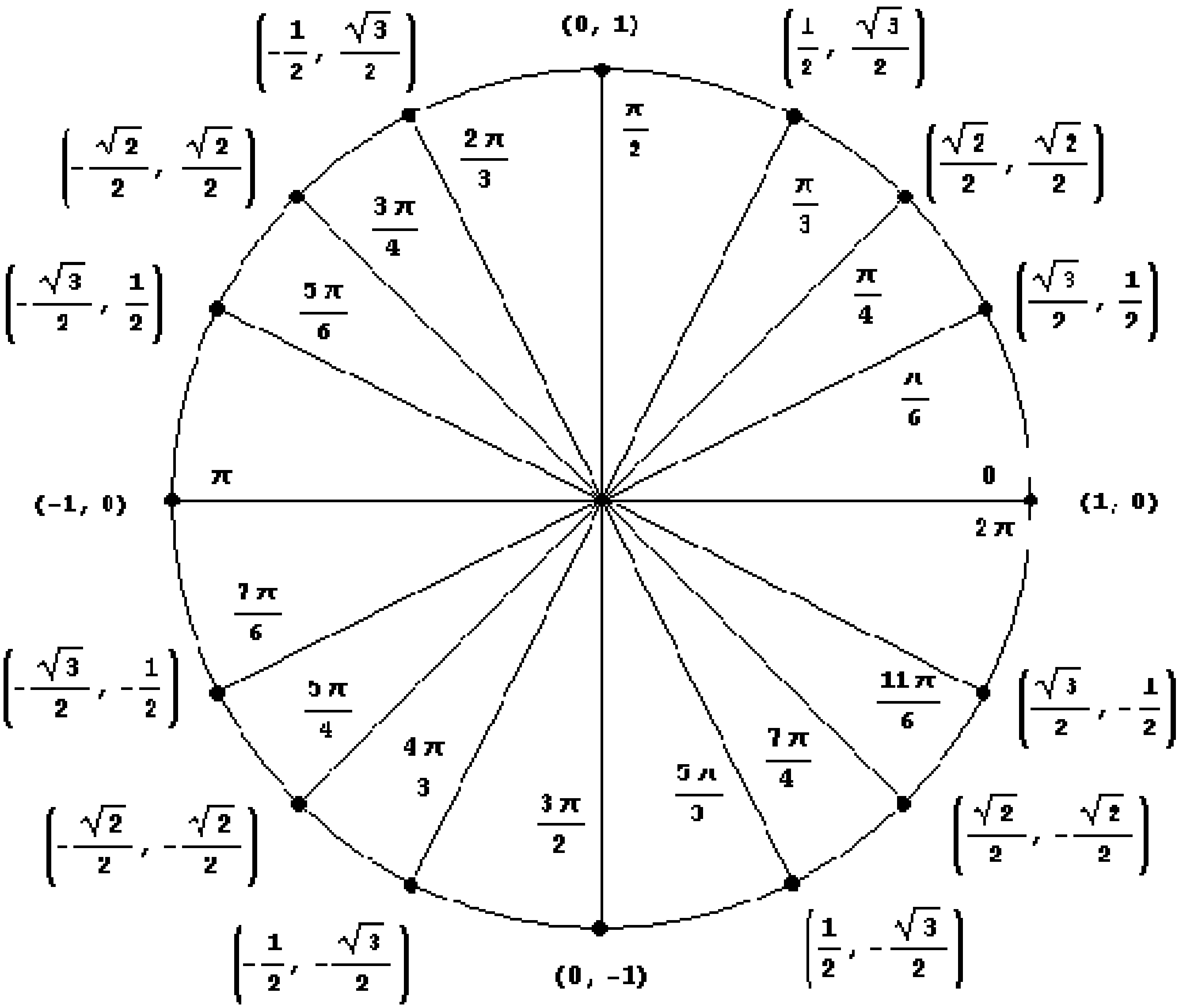
89. $\lim_{x \to 0} \frac{2}{\sin x} =$

<u>Circle with Radius 1</u>

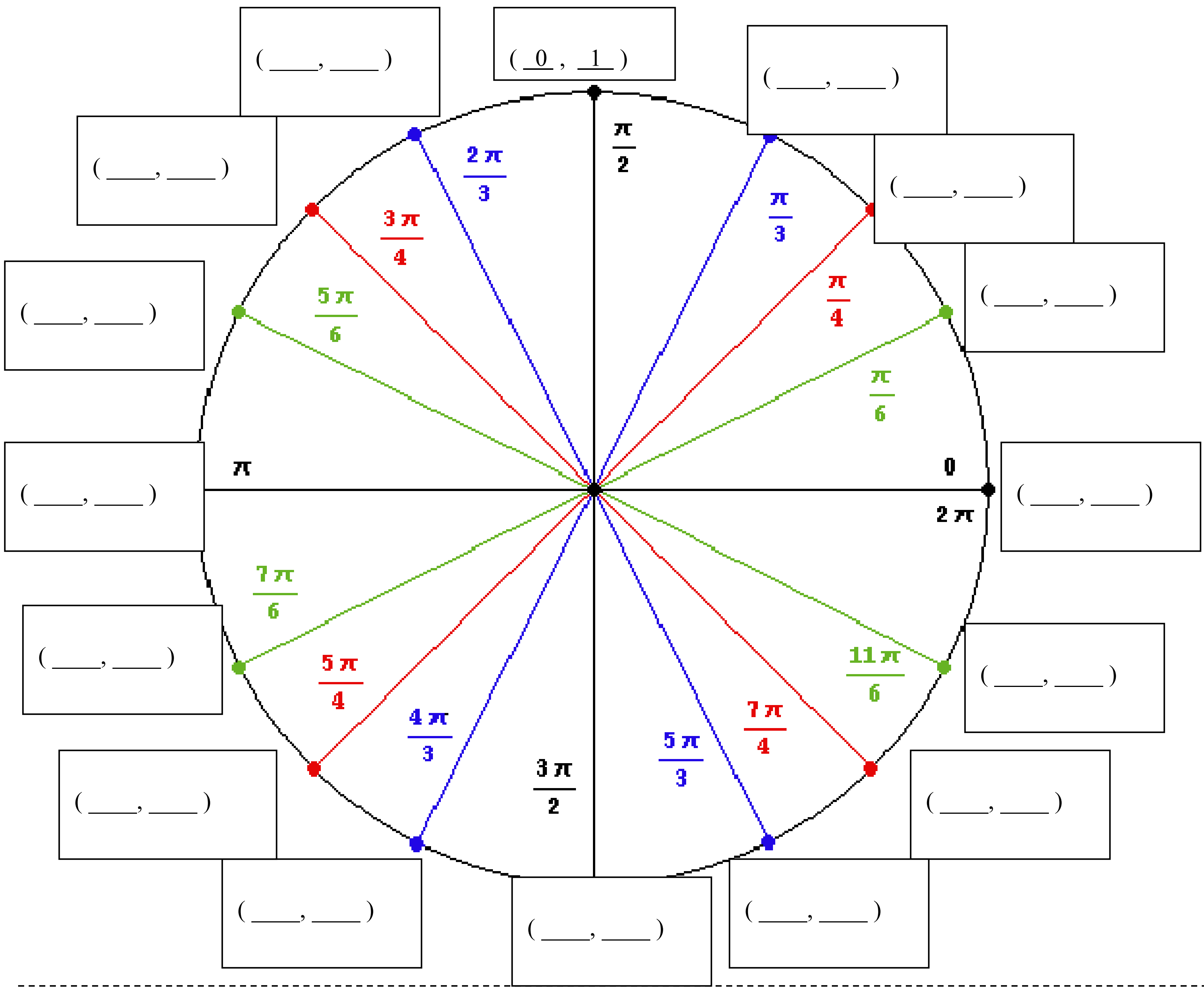
otherwise known as unit circle

$$(x,y) = (\cos\theta, \sin\theta)$$

You can determine the sine or cosine of an angle by using the unit circle. The x coordinate of the circle is cosine, and the y-coordinate is the sine.



Unit Circle Worksheet



DIRECTIONS: Please fill in the (x, y) coordinates above for the Unit Circle. Then proceed by filling in the values of the appropriate sin, cos and tan functions below. You will also receive another copy of this unit circle with your summer letter, so you can practice repeatedly.

sin (0) =	____________	sin (π/6) =	______________
cos(0) =	___________	cos (5π/6) =	_____________
sin(3π/2) =	____________	tan (3π/2) =	_____________
cos(π) =	____________	sin (π/4) =	_____________
sin (5π/2) =	____________	cos (11π/6) =	____________
tan (0) =	____________	sin (π) =	________________

Formula Sheet

Reciprocal Identities: $\csc x = \dfrac{1}{\sin x}$ $\sec x = \dfrac{1}{\cos x}$ $\cot x = \dfrac{1}{\tan x}$

Quotient Identities: $\tan x = \dfrac{\sin x}{\cos x}$ $\cot x = \dfrac{\cos x}{\sin x}$

Pythagorean Identities: $\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

Double Angle Identities: $\sin 2x = 2\sin x \cos x$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$$

Logarithms: $y = \log_a x$ is equivalent to $x = a^y$

Product property: $\log_b mn = \log_b m + \log_b n$

Quotient property: $\log_b \dfrac{m}{n} = \log_b m - \log_b n$

Power property: $\log_b m^p = p \log_b m$

Property of equality: If $\log_b m = \log_b n$, then m = n

Change of base formula: $\log_a n = \dfrac{\log_b n}{\log_b a}$

Derivative of a Function: Slope of a tangent line to a curve or the derivative: $\lim\limits_{h\to\infty} \dfrac{f(x+h)-f(x)}{h}$

Slope-intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

Standard form: Ax + By + C = 0